Expected Value and Variance Theorems

Let X be a discrete random variable with values $x_1, x_2, x_3, \ldots$ and associated probabilities $p_1, p_2, p_3, \ldots$.

Definition: The *expected value* or *mean* of the random variable X, which is denoted μ or $E(X)$, is defined as $\mu = E(X) = \sum_i x_i \cdot p_i$.

Theorems about expected value:

- $E(a \cdot X) = a \cdot E(X)$
- $E(X + c) = E(X) + c$
- $E(X \pm Y) = E(X) \pm E(Y)$
- If X and Y are independent random variables, $E(X \cdot Y) = E(X) \cdot E(Y)$
- $E(aX + bY + c) = a \cdot E(X) + b \cdot E(Y) + c$

Definition: The *variance* of the random variable X, which is denoted σ^2 or $V(X)$, is defined as $\sigma^2 = V(X) = E(X - \mu)^2 = \sum_i (x_i - \mu)^2 \cdot p_i$.

Definition: The *standard deviation* of the random variable X, which is denoted σ, is the positive square root of the variance.

Theorems about variance:

- $V(a \cdot X) = a^2 \cdot V(X)$
- $V(X + c) = V(X)$
- If X and Y are independent random variables, $V(X + Y) = V(X) + V(Y)$
- If X and Y are independent random variables, $V(X - Y) = V(X) + V(Y)$
- If X and Y are independent random variables, $V(a \cdot X + b \cdot Y + c) = a^2 \cdot V(X) + b^2 \cdot V(Y)$

Theorems about standard deviation:

- $\sigma_{a \cdot X} = |a| \cdot \sigma_X$
- $\sigma_{X+c} = \sigma_X$
- If X and Y are independent random variables, $\sigma_{X+Y} = \sqrt{\sigma_x^2 + \sigma_Y^2}$
- If X and Y are independent random variables, $\sigma_{X-Y} = \sqrt{\sigma_x^2 + \sigma_Y^2}$
- If X and Y are independent random variables, $\sigma_{aX+bY+c} = \sqrt{a^2\sigma_x^2 + b^2\sigma_Y^2}$

Geometric and Binomial Distributions

Geometric Probability Distribution

A random variable X has a **geometric distribution** if the following conditions exist:

- Each observation falls into one of two outcomes, often referred to as "success" and "failure."
- The observations are independent; that is, knowing the outcome of one trial tells you nothing about the outcome of other trials.
- The probability of success, which is denoted by *p*, is the same for each trial.
- X represents the number of the trial on which the first success occurs. So the possible values of X are integers 1, 2, 3, ...

$$P(X=k)=(1-p)^{k-1}\cdot p$$

Binomial Probability Distribution

A random variable X has a **binomial distribution** if the following conditions exist:

- There are a fixed number of trials or observations. This number is denoted by *n*.
- Each observation falls into one of two outcomes, often referred to as "success" and "failure."
- The *n* observations are independent; that is, knowing the outcome of one trial tells you nothing about the outcome of other trials.
- The probability of success, which is denoted by *p*, is the same for each trial.
- X represents the count or number of successes in the *n* trials. So the possible values of X are integers 0, 1, 2, . . ., *n*.

$$P(X=k)=\binom{n}{k}p^k(1-p)^{n-k}=\frac{n!}{k!\,(n-k)!}p^k(1-p)^{n-k}$$

Calculator Notes

The calculator has built-in functionality for computing geometric and binomial probabilities. Using the calculator for geometric computations seems unnecessary; however, *binompdf* - which calculates $P(X=k)$ - and *binomcdf* - which calculates $P(X\leq k)$ - are very useful.